Diffusion, relaxation, and response functions of solitons in one-dimensional antiferromagnets

1993 J. Phys.: Condens. Matter 5 7737

(http://iopscience.iop.org/0953-8984/5/41/020)

J. Phys.: Condens Matter 5 (1993) 7737–7750. Printed in the UK

Diffusion, relaxation, and response functions of solitons in one-dimensional antiferromagnets

B A Ivanov, A K Kolezhuk and E V Tartakovskaya

Theoretical Physics Division, Institute for Metal Physics, 36 Vernadsky Street, 252142 Kiev, Ukraine

Received 2 February 1993, in final form 20 May 1993

Abstract. Two mechanisms of diffusive kink motion in a quasi-one-dimensional rhombic antiferromagnet, placed in an external magnetic field, are considered, together with their effect on the soliton central peak in the dynamic structure factor (DSF). The temperature and field dependence of the diffusion coefficients is calculated. It is shown that non-dissipative diffusion gives zero contribution to the DSF, while ordinary Einstein diffusion changes its form considerably in the region of small wavevectors.

1. Introduction

Topological solitons of the kink type are known to play a special role in the dynamics and thermodynamics of quasi-one-dimensional magnets [1]; in particular, they make a peculiar contribution to the dynamic structure factor (DSF) which determines the cross section of inelastic neutron scattering [2]. This contribution has the form of a central peak (CP) in the DSF [3], *and the shape of the* CP *depends considerably on the character of the stochastic* thermal motion of kinks [4–6].

It should be noted that a non-Gaussian shape of CP was observed experimentally for the one-dimensional antiferromagnet (AFM) $(CH_3)_4NMnCl_3$ (TMMC) [4], but since the authors of [4] described their results on the basis of the exactly integrable sine–Gordon model, the analysis was made only in terms of anomalous non-dissipative diffusion.

Kink diffusion, caused by the interaction of kinks with the thermostat of quasilinear excitations (magnons in our case), was investigated for the first time by Wada and Schrieffer [7] for the scalar φ^4 model. They obtained a temperature dependence of the diffusion coefficient, $D \propto T^2$, which was criticized [8,9] as contradicting the Einstein relation $D\eta = T$ (D and η are the diffusion coefficient and viscous friction constant). Later it became clear [10,6] that there are two different mechanisms of soliton diffusion, which are characterized by two different diffusion coefficients: normal diffusion coefficient D (determined by the viscosity η through the relation $D = T/\eta$) and the anomalous non-dissipative diffusion coefficient D_* which is not connected with viscosity and is determined by shifts in the soliton coordinate occurring at random instants during collisions of the soliton with magnons. The temperature dependence $D_* \propto T^2$ is universal, in contrast to the temperature dependence of D, which is determined by the behaviour of $\eta = \eta(T)$. For exactly integrable systems such as the sine–Gordon model irreversible processes are absent, $\eta = 0$, and the only type of diffusion is the anomalous D_* type. If the integrability is violated (as in the case of double-sine–Gordon or the φ^4 model) and $\eta \neq 0$, the D_* diffusion is observed along with the normal D diffusion.

0953-8984/93/417737+14$07.50

In the present work, we shall analyse the two types of kink diffusion in AFMs by using a microscopic approach. It turns out that an external magnetic field H which is always present in experiments changes in principle the nature of viscosity and D diffusion in an easy-plane or rhombic AFM as compared to the case $H = 0$ considered earlier [11]. We shall analyse the influence of diffusion on the shape of the CP in AFMs and show that the role of the anomalous D_* diffusion in real magnets cannot be significant.

2. Model and kink solutions

Let us consider a model of a one-dimensional two-sublattice AFM that can be described by the following free-energy functional:

$$W = M_0^2 a^2 \int \mathrm{d}x \left\{ \frac{\delta}{2} \boldsymbol{m}^2 + \frac{\alpha}{2} (\nabla \boldsymbol{l})^2 - 2\boldsymbol{m}\boldsymbol{h} + w_\mathrm{a}(\boldsymbol{l}) \right\}. \tag{1}$$

Here $\boldsymbol{m}$ and $\boldsymbol{l}$ are the magnetization and antiferromagnetism vectors normalized by the condition $\boldsymbol{m}^2 + \boldsymbol{l}^2 = 1$; M_0 is the value of equilibrium magnetization of a sublattice; δ and α are the constants of uniform and non-uniform exchange, respectively (the exchange field is $H_\mathrm{e} = \delta M_0/4$); $\boldsymbol{h} = \boldsymbol{H}/M_0$; $\boldsymbol{H}$ is an external magnetic field, and a is the lattice constant. The anisotropy energy $w_\mathrm{a}(\boldsymbol{l})$ will be chosen in the form typical of a rhombic AFM:

$$w_\mathrm{a} = \tfrac{1}{2}\beta_1 l_x^2 + \tfrac{1}{2}\beta_2 l_y^2 \qquad \beta_1 > \beta_2 > 0 \tag{2}$$

the z axis being the easiest. In the case of an easy-plane AFM of the TMMC type, the constant $\beta_2 \ll \beta_1$ determines the weak anisotropy in the basal plane yz. For an AFM with a clearly manifested rhombic anisotropy of the $CsMnCl_3{\cdot}2H_2O$ type, the constants β_1 and β_2 have the same order of magnitude.

It is well known [12] that the Landau–Lifshits equations corresponding to the functional (1) can be reduced to a single equation for the vector $\boldsymbol{l}$, the total magnetization vector $\boldsymbol{m}$ being connected with $\boldsymbol{l}$, $\partial \boldsymbol{l}/\partial t$ through the following relation:

$$2\delta \boldsymbol{m} = (1/gM_0)[\boldsymbol{l} \times (\partial \boldsymbol{l}/\partial t)] + \boldsymbol{h} - \boldsymbol{l}(\boldsymbol{h}\cdot\boldsymbol{l}). \tag{3}$$

Here $g = 2\mu_0/\hbar$ and μ_0 is the magnitude of Bohr's magneton. The effective equation of motion for the vector $\boldsymbol{l}$ (which can be regarded as a unit vector to a fairly high degree of accuracy, $\boldsymbol{l}^2 = 1$) can be obtained from the Lagrangian

$$L = M_0^2 a^2 \int \mathrm{d}x \left\{ \frac{\alpha}{2}\left[\frac{1}{c^2}\left(\frac{\partial \boldsymbol{l}}{\partial t}\right)^2 - (\nabla \boldsymbol{l})^2 \right] + \frac{4}{g\delta M_0}\boldsymbol{h}\left(\frac{\partial \boldsymbol{l}}{\partial t} \times \boldsymbol{l}\right) - \frac{2}{\delta}(\boldsymbol{l}\cdot\boldsymbol{h})^2 - w_\mathrm{a}(\boldsymbol{l}) \right\} \tag{4}$$

where $c = gM_0\sqrt{\alpha\delta}/2$ is the limiting velocity of kinks at $H = 0$. For $H = 0$, this Lagrangian describes an anisotropic generalization of the well known chiral σ model and is Lorentz invariant. However, in the presence of a magnetic field, the Lagrangian acquires terms linear in $\partial \boldsymbol{l}/\partial t$ which destroy the Lorentz invariance.

Writing $\boldsymbol{l}$ in terms of angular variables, $l_z = \cos\theta$, $l_x + \mathrm{i}l_y = \sin\theta \exp(\mathrm{i}\varphi)$, we obtain the following equations of motion for the angles θ and φ:

$$\begin{aligned}
&\alpha[\partial^2\theta/\partial x^2 - (1/c^2)\partial^2\theta/\partial t^2] - \sin\theta\cos\theta[\alpha(\partial\varphi/\partial x)^2\bar{\beta}_1\cos^2\varphi - \bar{\beta}_2\sin^2\varphi] \\
&\qquad - (8h/g\delta M_0)F(\theta,\varphi)\partial\varphi/\partial t = 0 \\
&\alpha\{(\partial/\partial x)[(\sin^2\theta)\partial\varphi/\partial x] - (1/c^2)(\partial/\partial t)[(\sin^2\theta)\partial\varphi/\partial t]\} + (\bar{\beta}_1 - \bar{\beta}_2)\sin^2\theta\sin\varphi\cos\varphi \\
&\qquad + (8h/g\delta M_0)F(\theta,\varphi)\partial\theta/\partial t = 0.
\end{aligned} \tag{5}$$

Here $\bar{\beta}_1$ and $\bar{\beta}_2$ are effective anisotropy constants, which are determined by the renormalization of the anisotropy energy: $w_a \to \bar{w}_a = w_a + 2(\boldsymbol{h}\cdot\boldsymbol{l})^2/\delta$. The form of the function $F(\theta,\varphi)$ depends on the field orientation. We shall seek the soliton solution of system (5) in the form of a travelling wave with a constant velocity v: $\theta = \theta(\xi), \varphi = \varphi(\xi)$, where $\xi = (x - vt)(1 - v^2/c^2)^{-1/2}$. In this case, (5) can be written in the form

$$\begin{gathered}\alpha\theta'' - \alpha\sin\theta\cos\theta(\varphi')^2 - \sin\theta\cos\theta(\bar{\beta}_1\cos^2\varphi + \bar{\beta}_2\sin^2\varphi) \\ + [8hvF(\theta,\varphi)/g\delta M_0(1 - v^2/c^2)^{1/2}]\varphi' = 0 \\ \alpha(\varphi'\sin^2\theta)' + (\bar{\beta}_1 - \bar{\beta}_2)\sin^2\theta\sin\varphi\cos\varphi - [8hvF(\theta,\varphi)/g\delta M_0(1 - v^2/c^2)^{1/2}]\theta' = 0\end{gathered} \tag{6}$$

(the prime indicates differentiation with respect to ξ).

We shall confine our analysis to the case closest to the experimental situation, when the field is directed along the y axis. In this case, $\bar{\beta}_1 = \beta_1$, $\bar{\beta}_2 = \beta_2 + 4h^2/\delta$, $F(\theta,\varphi) = \sin^2\theta\sin\varphi$. In the static case $v = 0$, there exist solutions of system (6) describing two types of kink. In one type (zy kink), the rotation of the vector $\boldsymbol{l}$ takes place in the easy plane zy (i.e., $\varphi = \pm\pi/2$), while in the other (zx kink) the vector $\boldsymbol{l}$ rotates through the difficult axis x ($\varphi = 0, \pi$). The behaviour of the angle θ in both kinks is described by the solution of

$$\cos\theta_0 = \pm\tanh(\xi/x_0) \tag{7}$$

where $x_0 = (\alpha/\bar{\beta}_2)^{1/2}$ for the zy kink and $x_0 = (\alpha/\bar{\beta}_1)^{1/2}$ for the zx kink. If $\bar{\beta}_2 < \bar{\beta}_1$, i.e., $H < H_c$, $H_c = \frac{1}{2}M_0[(\beta_1 - \beta_2)\delta]^{1/2}$, the zy kink is preferable from the energy point of view for $v = 0$, while for $H > H_c$ the zx kink becomes more advantageous.

However, for $v \neq 0$, the only solution 'surviving' without changes (except the Lorentz contraction of the kink) is that with $\varphi = 0, \pi$. The kink with $\varphi = \pm\pi/2$ is modified since $F(\theta, \pi/2) \neq 0$, and the vector $\boldsymbol{l}$ leaves the easy plane.

As was shown earlier [13, 14], the modification of the zy kink at non-zero velocity can be approximately described as a turn of the plane in which the vector $\boldsymbol{l}$ rotates, i.e., the soliton solution has a form of (7) with $\varphi = \varphi_0(v)$, and $x_0 = [\alpha/\bar{\beta}(\varphi_0)]^{1/2}$, where $\bar{\beta}(\varphi_0) = \bar{\beta}_2 + (\bar{\beta}_1 - \bar{\beta}_2)\cos^2\varphi_0(v)$. At some critical velocity, $v = v_c$, $\varphi_0(v_c) = 0$ or π, and a kink of zy type is continuously transformed into a kink of zx type. The critical velocity v_c is given by [15, 13]

$$v_c = c|\bar{\beta}_2 - \bar{\beta}_1|\sqrt{\delta}[(\pi h)^2\bar{\beta}_1 + \delta(\bar{\beta}_2 - \bar{\beta}_1)^2]^{-1/2} \tag{8}$$

and for values of the field that are not close to H_c, the velocities v_c and c are of the same order of magnitude. In our further consideration we shall assume that $H \ll H_c$, and that the temperature is low enough for the mean thermal velocity of kinks to be small as compared to v_c. This will allow us to disregard the effects of modification of kink structure.

3. Generalized Langevin equation for Brownian motion of kinks

Before we proceed to the calculation of kinetic coefficients for solitons, it is useful to make some remarks. The peculiarity of antiferromagnets consists in that the potential of a kink is non-reflective for magnons in the case of rhombic anisotropy of type (2) for $H = 0$, and hence scattering processes do not make any contribution to η [11], and the value of η is determined by three-magnon processes, $\eta = \eta_3 \propto (T/E_0)^2$, where E_0 denotes a kink rest energy. The introduction of small corrections to the anisotropy energy $w_a = w_a^{\text{rhomb}} + \varepsilon \Delta w_a$, $\varepsilon \ll 1$, leads to the emergence of the two-magnon viscosity $\eta_2 \propto \varepsilon^2 T/E_0$, and hence the behaviour of the total viscosity $\eta = \eta_2 + \eta_3$ depends on the relation between small parameters ε and T/E_0: $\eta \propto T^2$ for $T > \varepsilon^2 E_0$, and $\eta \propto T$ for $T < \varepsilon^2 E_0$. For example, in [11] the fourth-order anisotropy was taken for the 'correction' Δw_a. We shall prove, however, that the presence of an external magnetic field also leads to two-magnon processes, and for a certain geometry of the problem the relevant correction may prove to be significant. The 'weight' of the two-magnon viscosity can be varied by changing the field, which can obviously give additional information in neutron scattering experiments.

Let us consider the motion of an individual zy-type kink (only this type is stable at low velocities for $H \ll H_c$) in the magnon thermostat (we disregard the kink–kink interactions, assuming that the soliton gas is rarefied). In order to analyse such a motion, we return to the Lagrangian (4). In angular variables, the Lagrangian has the form

$$L = M_0^2 a^2 \int \mathrm{d}x \left\{ \frac{\alpha}{2c^2}[\dot{\theta}^2 + \sin^2\theta \dot{\varphi}^2] - \frac{\alpha}{2}[(\nabla\theta)^2 + \sin^2\theta(\nabla\varphi)^2] - \frac{1}{2}\sin^2\theta[\bar{\beta}_1 \cos^2\varphi + \bar{\beta}_2 \sin^2\varphi] - \frac{8h}{g\delta M_0}\sin^2\theta\cos\varphi\dot{\theta} \right\}. \tag{9}$$

In order to construct the Hamiltonian describing the 'kink+magnons' system, we shall use the following *ansatz*:

$$\theta = \theta_0(x - x_s) + \vartheta(x,t) \qquad \varphi = \varphi_0 + (1/\sin\theta_0)\mu(x,t). \tag{10}$$

Here $x_s = x_s(t)$ is the kink coordinate and θ_0 and φ_0 correspond to static soliton solutions of the system of equations (5). In other words, θ_0 is described by formula (7) for $v = 0$, $\varphi_0 = \pm\pi/2$. The function $\vartheta(x,t)$ is subjected to the constraint $\int \mathrm{d}x \vartheta \nabla\theta_0 = 0$, which excludes the zero-frequency (translational) mode corresponding to the motion of the kink from the magnon field ϑ.

It should be noted that the expansion in the neighbourhood of static solutions is applicable only in the velocity region where the kink solution is modified weakly, i.e., in the region $|\dot{x}_s| \ll v_c$. Consequently, an analysis of the Brownian motion of solitons carried out in this way will be correct only if the mean thermal velocity v_T of a kink is small as compared to the critical velocity v_c, i.e., under the condition

$$T/E_0 \ll \{1 + [\pi^2\beta_1/4(\beta_1 - \beta_2)](H^2/H_c^2)/(1 - H^2/H_c^2)^2\}^{-1}.$$

Therefore, we can describe diffusion by using this approach only for $T \ll E_0$ and for values of the field differing considerably from the critical value H_c.

Substituting (10) into the Lagrangian (9), we can write $L = E_0 + m_* \dot{x}_s^2/2 + L_1 + L_2 + \ldots$, where L_n contains magnon fields ϑ and μ to the total power n; here $E_0 = 2M_0^2 a^2 [\alpha\bar{\beta}_2]^{1/2}$ is the rest energy of a kink, and $m_* = E_0/c^2$ its effective mass. It can be proved that the one-magnon term

$$L_1 = \sin\varphi_0 \frac{h}{(\bar{\beta}_2\delta)^{1/2}} \frac{2E_0}{c} \int \mathrm{d}x\, \mu\dot{\theta}_0 \sin\theta_0 + \frac{\alpha M_0^2 a^2}{c^2} \int \mathrm{d}x\, \theta_0 \dot{\vartheta}$$

leads to a renormalization of the static kink solution (θ_0, φ_0) corresponding to its modification at non-zero velocity. Since this modification can be neglected in the velocity region $|\dot{x}_s| \ll v_c$ we are interested in, L_1 will be disregarded in further analysis.

In order to make the formalism more clear and avoid overloading our consideration with cumbersome expressions, we shall give the derivation of effective equations of motion for a soliton, taking into account the field-induced two-magnon processes only. The three-magnon processes will be analysed later, along with the contribution of higher-order terms in anisotropy energy.

The two-magnon Lagrangian L_2 can be written in the form $L_2 = L_2^{(0)} + \Delta L_2^{(\mathrm{int})}$, where $L_2^{(0)}$ is defined as

$$L_2^{(0)} = \frac{E_0}{4x_0} \int \mathrm{d}x \left\{ \frac{1}{\omega_0^2} [\dot{\vartheta}^2 + \dot{\mu}^2] - \vartheta \hat{\mathcal{H}}_0 \vartheta - \mu(\hat{\mathcal{H}}_0 + p)\mu \right\} \tag{11}$$

and can be diagonalized [16]. Here $x_0 = [\alpha/\bar{\beta}_2]^{1/2}$ is the thickness of a kink, $\omega_0 = c/x_0$ the characteristic magnon frequency, and the quantity $p = |\bar{\beta}_1 - \bar{\beta}_2|/\bar{\beta}_2$ has the meaning of the rhombicity parameter (which depends on the strength H of the field). The well known operator $\hat{\mathcal{H}}_0 = -x_0^2 \partial^2/\partial x^2 + 1 - 2\cosh^{-2}[(x - x_s)/x_0]$ with a reflectionless potential has eigenfunctions ψ_k corresponding to magnons of the continuous spectrum:

$$\begin{aligned} &\psi_k = \{(\tanh z - \mathrm{i}kx_0)/[L(1 + k^2 x_0^2)]^{1/2}\} \exp(\mathrm{i}kx) \qquad z = (x - x_s)/x_0 \\ &\hat{\mathcal{H}}_0 \psi_k = (1 + k^2 x_0^2)\psi_k \end{aligned} \tag{12}$$

and also the zero-frequency mode $\psi_0 = (2x_0)^{1/2} \cosh^{-1}[(x - x_s)/x_0] \propto \theta_0'$, corresponding to a magnon localized at a kink in the case of the μ field.

The Lagrangian $\Delta L_2^{(\mathrm{int})}$ describing the two-magnon interaction has the form

$$\Delta L_2^{(\mathrm{int})} = \frac{2E_0}{c} \frac{h}{[\bar{\beta}_2 \delta]^{1/2}} \int \mathrm{d}x [-\vartheta \dot{\mu} \sin\theta_0 \sin\varphi_0 + O(\dot{x}_s)]. \tag{13}$$

A further analysis (using the perturbation theory) requires knowledge of the small parameters of the problem. While using the expansion in the neighbourhood of the static soliton solution (10), we have practically assumed that the kink velocity $\dot{x}_s$ is small (although the 'actual' small parameter in this case is the ratio v_T/v_c; see above). Besides, we have to assume the quantity $\varepsilon \equiv h/[\bar{\beta}_2\delta]^{1/2}$ to be small for the interaction $\Delta L_2^{(\mathrm{int})}$ to be regarded as weak. (It should be noted that the condition $\varepsilon \ll 1$ is not always satisfied. For example, in the case of an easy-plane magnet of the TMMC type, we have $\beta_2 \simeq 0$, $\bar{\beta}_2 = 4h^2/\delta$, and $\varepsilon = \frac{1}{2}$, irrespective of the magnitude of the field. This case cannot be analysed using our approach, but we believe that the qualitative results will be the same.)

Let us expand the magnon field in the eigenfunctions of the operator $\hat{\mathcal{H}}_0$:

$$\vartheta = \sum_k q_k \psi_k \qquad \mu = Q_0\psi_0 + \sum_k Q_k \psi_k \tag{14}$$

and go over to the Hamilton formalism in the magnon variables. Presenting the coordinates q and Q and the momenta p and P conjugate to them in terms of Bose amplitudes (corresponding to the creation and annihilation operators in the quantum theory),

$$\begin{aligned} &q_k = (\hbar x_0 \omega_0^2/E_0\omega_k)^{1/2}[a_{-k}^* + a_k] \qquad p_k = \mathrm{i}(\hbar E_0 \omega_k/4x_0\omega_0^2)^{1/2}[a_k^* - a_{-k}] \\ &Q_{0,k} = (\hbar x_0 \omega_0^2/E_0\Omega_{0,k})^{1/2}[A_{0,-k}^* + A_{0,k}] \qquad P_{0,k} = \mathrm{i}(\hbar E_0 \Omega_{0,k}/4x_0\omega_0^2)^{1/2}[A_{0,k}^* - A_{0,-k}] \end{aligned} \tag{15}$$

we obtain the Hamilton function $H = P_0\dot{Q}_0 + \sum_k(p_k\dot{q}_k + P_k\dot{Q}_k) - L$ (to be more precise, H is the Routh function since x_s remains a Lagrangian variable) in the form

$$H = -m_*\dot{x}_s^2/2 + H_0 + H_{\rm int} \tag{16}$$

where H_0 describes the gas of non-interacting magnons against the background of a kink:

$$H_0 = \sum_k \hbar\omega_k a_k^* a_k + \sum_k \hbar\Omega_k A_k^* A_k + \hbar\Omega_0 A_0^* A_0. \tag{17}$$

The spectrum of the system contains magnons of three types: two branches of the continuous spectrum (θ and φ magnons) with the frequencies

$$\omega_k = \omega_0[1 + k^2x_0^2]^{1/2} \qquad \Omega_k = \omega_0[1 + p + k^2x_0^2]^{1/2} \tag{18}$$

and a mode of φ magnons localized at a kink and having the frequency $\Omega_0 = \omega_0\sqrt{p}$. The interaction is described by the Hamiltonian $H_{\rm int}$ which can be expanded into a power series in the amplitudes A_k, a_k: $H_{\rm int}^{(1)} + H_{\rm int}^{(2)} + \ldots$. The one-magnon term $H_{\rm int}^{(1)}$ can be eliminated by using the 'shift' transformation of magnon amplitudes [17], which leads to various renormalizations of the static kink solution (see above). Since these corrections are of second order in the small parameters, they will simply be omitted in the further analysis.

The two-magnon Hamiltonian $H_{\rm int}^{(2)}$ describes the scattering of magnons by a kink. In the first order in small parameters ε and $\dot{x}_s$ it can be written in the form $H_{\rm int}^{(2)} = \dot{x}_s T + \varepsilon U$, where the first term appears as we go over from $L_2^{(0)}$ to H_0 due to an explicit dependence of the wavefunctions ψ_k on the kink coordinate x_s:

$$T = \sum_{12}(T'_{12}a_1^*a_2 + T''_{12}A_1^*A_2) + \frac{1}{2}\sum_{12}(\tilde{T}'_{12}a_1^*a_2^* + \tilde{T}''_{12}A_1^*A_2^* + {\rm cc}). \tag{19}$$

It will be shown later that this term corresponds to the processes occurring without a momentum transfer and leading only to an asymptotic shift in the kink coordinate, while the second term is associated with the Lagrangian $\Delta L_2^{(\rm int)}$ and describes inelastic scattering processes:

$$U = \sum_{12}\left\{U_{12}(A_1^*a_2 + A_1^*a_{-2}^*) + {\rm cc}\right\} + \sum_k \left\{U_{0k}A_0^*a_k - U_{0k}^*A_0^*a_k^* + {\rm cc}\right\}. \tag{20}$$

The matrix elements of the operator T are defined as (the index $1 \equiv k_1$ everywhere, etc):

$$\begin{aligned} T'_{12} &= -{\rm i}\hbar\frac{\omega_1+\omega_2}{2\sqrt{\omega_1\omega_2}}\int {\rm d}x\left[\psi_1^*\frac{\partial\psi_2}{\partial x_s}\right]\bigg|_{x_s=0} \exp[{\rm i}(k_2-k_1)x_s] \\ \tilde{T}'_{12} &= [(\omega_1-\omega_2)/(\omega_1+\omega_2)]T'_{1,-2} \end{aligned} \tag{21}$$

and T''_{12} and $\tilde{T}''_{12}$ are given by similar expressions in which the frequencies $\omega_{1,2}$ are replaced by $\Omega_{1,2}$; we must assume that the indices 1 and 2 label not only the modes of the continuous spectrum, but also the localized mode. For the matrix elements of the operator U, we have

$$\begin{aligned} U_{12} &= -2{\rm i}\hbar\omega_0\sin\varphi_0(\Omega_1/\omega_2)^{1/2}[\![\{\pi x_0[1+(k_1x_0)^2+(k_2x_0)^2]/L(1+k_1^2x_0^2)^{1/2} \\ &\quad\times(1+k_2^2x_0^2)^{1/2}\}/\cosh[\pi(k_1-k_2)x_0/2]]\!]\exp[{\rm i}(k_2-k_1)x_s] \\ U_{0k} &= -2\hbar\omega_0\sin\varphi_0(\Omega_0x_0/2L\omega_k)^{1/2}[\pi k^2x_0^2/\sinh(\pi kx_0/2)]\exp({\rm i}kx_s). \end{aligned} \tag{22}$$

The terms of A^*a^* type give no contribution to the viscosity [18], and we shall not write the relevant amplitudes.

Equations of motion in the variables a_k, A_k, a_0, and x_s have the form

$$(\partial/\partial t)\partial H/\partial \dot{x}_s - \partial H/\partial x_s = 0 \qquad i\hbar \dot{a}_k = \partial H/\partial a_k^* \qquad i\hbar \dot{A}_{0,k} = \partial H/\partial A_{0,k}^*.$$

Taking into account the remarks made above, we obtain the following coupled system of equations for the kink coordinate and magnon amplitudes:

$$\begin{aligned}
m_* \ddot{x}_s = F(t) &= \sum_{12}[\mathrm{i}(\omega_1-\omega_2)T'_{12}a_1^*a_2+\mathrm{i}(\Omega_1-\Omega_2)T''_{12}A_1^*A_2]+\varepsilon\sum_{12}[\mathrm{i}(k_1-k_2)U_{12}A_1^*a_2+\mathrm{CC}]\\
&\quad+\varepsilon\sum_k[-\mathrm{i}kU_{0k}A_0^*a_k+\mathrm{CC}]\\
\mathrm{i}\hbar\dot{a}_k &= \hbar\omega_k a_k + \sum_{k'}\{\dot{x}_s T'_{kk'}a_{k'} + \varepsilon U^*_{k'k}A_{k'}\} + \varepsilon U^*_{0k}A_0 \\
\mathrm{i}\hbar\dot{A}_k &= \hbar\Omega_k A_k + \sum_{k'}\{\dot{x}_s T''_{kk'}A_{k'} + \varepsilon U_{kk'}a_{k'}\}\\
\mathrm{i}\hbar\dot{A}_0 &= \hbar\Omega_0 A_0 + \varepsilon\sum_k U_{0k}a_k.
\end{aligned} \tag{23}$$

This system can be investigated by using the standard thermodynamic perturbation theory (see, e.g., [19]). For this purpose, we shall go over to amplitudes in the 'interaction representation' ($\tilde{a}_k(t) = a_k(t)\exp(\mathrm{i}\omega_k t)$, etc) and transform equations (23) into integral equations. Using the assumption about the adiabatic initiation of interaction, we obtain initial conditions of the type $\tilde{a}_k(t) \to a_{k0}$ at $t \to -\infty$, a_{k0} =constant. Solving the integral equations by using the iteration method with such initial conditions, we obtain $\tilde{a}_k(t)$ as a function of $\{a_{k0}\}$ and t in the form of a series in the small parameters ε and $\dot{x}_s$. The averaging over the thermostat at $t = -\infty$ (i.e., over $\{a_{k0}\}$) is determined by the equilibrium Gibbs distribution function $f \propto \exp(-H_0/T)$, so that $\langle a^*_{k0}a_{k'0}\rangle = (T/\hbar\Omega_k)\delta_{kk'}$, etc. It should be noted that the simplest quantum-mechanical modification of these formulae can be carried out by replacing the classical multipliers $T/\hbar\omega_k$, $T/\hbar\Omega_k$ by the Bose occupational numbers $n_k = [\exp(\hbar\omega_k/T)-1]^{-1}$, $N_k = [\exp(\hbar\Omega_k/T)-1]^{-1}$, or n_k+1, N_k+1; the choice of the version is obvious. In this way, we can generalize our analysis to the case of low temperatures $T \ll \hbar\omega_0$. The perturbation theory constructed by such a method makes it possible to calculate average values over magnon amplitudes in any order of ε and $\dot{x}_s$.

In zeroth approximation in the small parameters, the average value of the force $F(t)$ acting on a kink is equal to zero, and its two-point correlator has the form

$$\begin{aligned}
\langle F(t)F(0)\rangle^{(0)} &= \sum_{12}(\omega_1-\omega_2)^2|T'_{12}|^2 n_1(N_2+1)\exp[\mathrm{i}(\omega_1-\omega_2)t-\mathrm{i}(k_1-k_2)\Delta x_s(t)]\\
&\quad+\sum_{12}(\Omega_1-\Omega_2)^2|T''_{12}|^2 N_1(N_2+1)\exp[\mathrm{i}(\Omega_1-\Omega_2)t-\mathrm{i}(k_1-k_2)\Delta x_s(t)]\\
&\quad+\sum_{12}(k_1-k_2)^2|\varepsilon U_{12}|^2[N_1(n_2+1)+(N_1+1)n_2]\\
&\quad\times\exp[\mathrm{i}(\Omega_1-\omega_2)t-\mathrm{i}(k_1-k_2)\Delta x_s(t)]\\
&\quad+\sum_k k^2|\varepsilon U_{0k}|^2[N_0(n_k+1)+(N_0+1)n_k]\exp[\mathrm{i}(\Omega_0-\omega_k)t+\mathrm{i}k\Delta x_s(t)]
\end{aligned} \tag{24}$$

where $\Delta x_s(t) \equiv x_s(t) - x_s(0)$ is the kink displacement during the time t. In the first order of the perturbation theory, the mean force becomes non-zero and can be expressed in terms of the zeroth-order correlation function:

$$\langle F(t)\rangle^{(1)} = -\frac{1}{T}\int_{-\infty}^{t} \mathrm{d}t' \langle F(t-t')F(0)\rangle^{(0)} \dot{x}_s(t') \tag{25}$$

(this relation is essentially one of the forms of the fluctuation-dissipation theorem). Thus, we can single out in the force $F(t)$ the two components, viz., the regular 'slow' component $\langle F(t)\rangle^{(1)}$ and the rapidly fluctuating 'random force' $f(t) = F(t) - \langle F(t)\rangle^{(1)}$. This leads to the following generalized Langevin equation for the kink coordinate:

$$m_* \ddot{x}_s = -\int_{-\infty}^{t} \mathrm{d}\tau\, \gamma(t-\tau)\dot{x}_s(\tau) + f(t) \tag{26}$$

where $\gamma(\tau) = (1/T)\langle f(t) f(0)\rangle$ plays the role of the memory function, and the random force correlator $\langle f(t)f(0)\rangle$ can obviously be regarded as coinciding with (24) to within $\dot{x}_s^2$. (It should be noted that a similar equation was written in [10] by using the Mori method for solitons in the φ^4 model.)

Let us consider the contribution of various interactions to the regular component of the force acting on a kink in greater detail. For this purpose, we shall analyse the function $\Gamma(\omega) = \pi\tilde{\gamma}(\omega) - \mathrm{i}\mathcal{P}\int \mathrm{d}\omega'\,(1/\omega')\tilde{\gamma}(\omega-\omega')$, where $\tilde{\gamma}(\omega)$ is the Fourier transform of the memory function and $\mathcal{P}$ is the value principal. The quantity $\Gamma(\omega)$ determines the solution of the Langevin equation (26) and can be interpreted as the 'coefficient of friction', which depends on the frequency. It can be easily seen that the expansion of $\Gamma(\omega)$ in the region of low frequencies $\omega \ll \omega_0$ has the form

$$\Gamma(\omega) = \eta + \mathrm{i}\omega\Delta m_* + \lambda\omega^2 + \ldots \tag{27}$$

and the characteristic frequency at which $\Gamma(\omega)$ decreases considerably has the order of the magnon frequency ω_0. The expansion coefficients in (27) have a simple physical meaning. The quantity $\eta = \Gamma(0)$ is the viscosity which is determined by the inelastic scattering of magnons at a kink:

$$\begin{aligned} &\eta = \eta_{\mathrm{I}} + \eta_{\mathrm{II}} \\ &\eta_{\mathrm{I}} = \frac{\pi}{T}\sum_k k^2 |\varepsilon U_{0k}|^2 [N_0(n_k+1) + n_k(N_0+1)]\delta(\Omega_0 - \omega_k) \\ &\eta_{\mathrm{II}} = \frac{\pi}{T}\sum_{12} (k_1-k_2)^2 |\varepsilon U_{12}|^2 [N_1(n_2+1) + n_2(N_1+1)]\delta(\Omega_1 - \omega_2) \end{aligned} \tag{28}$$

(it should be recalled that we are speaking here only of the two-magnon viscosity η_2 making the main contribution to the total viscosity at low temperatures $T < \varepsilon^2 E_0$). The viscosity η determines irreversible processes occurring in the system and is connected with the 'normal' diffusion coefficient D through the Einstein relation $D = T/\eta$.

In contrast, the coefficient λ is not associated with dissipation and is due to scattering processes without a momentum transfer:

$$\lambda = \frac{\pi}{T}\sum_{12} \{|T'_{12}|^2 n_1(n_2+1)\delta(\omega_1-\omega_2) + |T''_{12}|^2 N_1(N_2+1)\delta(\Omega_1-\Omega_2)\}. \tag{29}$$

It determines the non-Einsteinian (non-dissipative) diffusion, the quantity $D_* = T\lambda/m_*^2$ playing the role of the coefficient of this diffusion [20].

Finally, the term in (27) that is linear in frequency simply leads to the replacement of the kink mass m_* by the effective 'dynamic' mass $\bar{m}_*$:

$$\bar{m}_* = m_* + \Delta m_* = m_* + T^{-1}\sum_{12}[|T'_{12}|^2 n_1(n_2+1) + |T''_{12}|^2 N_1(N_2+1)]. \tag{30}$$

Let us calculate the values of D_* and η characterizing the stochastic thermal motion of a kink. The coefficient D_* can be easily calculated by using explicit forms of relevant amplitudes; for both types of kink, we have

$$D_* = \begin{cases} (2/\pi)(\hbar T/m_* E_0)\{\exp[-\hbar\omega_0/T] + \exp[-\hbar\omega_0(1+p)^{1/2}/T]\} \\ \qquad T \ll \hbar\omega_0 \\ (2/3\pi)\omega_0 x_0^2(T/E_0)^2 + (2/\pi)(\hbar T/E_0 m_*)\exp[-\hbar\omega_0(1+p)^{1/2}/T] \\ \qquad \hbar\omega_0 \ll T \ll \hbar\omega_0(1+p)^{1/2} \\ (2/3\pi)\omega_0 x_0^2(T/E_0)^2[1+(1+p)^{-1/2}] \\ \qquad T \gg \hbar\omega_0(1+p)^{1/2}. \end{cases} \tag{31}$$

The two-magnon viscosity constant determined by the external field differs from zero only for a zy kink, i.e., is manifested for $H < H_c$. (It should be recalled that the calculations are carried out for the case $v \ll v_c$, i.e., we must assume that H is not close to H_c.) The two-magnon viscosity is $\eta_2 = \eta_I + \eta_{II}$ (see (28)). The contribution of processes involving the localized magnon η_I differs from zero only for $p > 1$:

$$\eta_I = \varepsilon^2(\hbar^2\omega_0/Tx_0^2)[(p-1)^{5/2}/p^{1/2}]\pi^2/\sinh^2[\pi\sqrt{p-1}/2]\sinh^2[\hbar\omega_0\sqrt{p}/2T]. \tag{32}$$

The calculation of the second component η_{II} associated with a mutual conversion of magnons from different branches of the continuous spectrum is a more complicated problem. The calculation gives

$$\eta_{II} = \begin{cases} \varepsilon^2(\hbar/x_0^2)(\hbar\omega_0/T)^{1/2}[(2\pi)^{3/2}p^{1/2}(1+p)^{3/4}/\cosh^2(\pi\sqrt{p}/2)] \\ \times\exp(-\hbar\omega_0\sqrt{1+p}/T) \qquad T \ll \hbar\omega_0(1+p)^{1/2} \\ (2\pi T\varepsilon^2/\omega_0 x_0^2)f_2(p) \qquad T \gg \hbar\omega_0(1+p)^{1/2} \end{cases} \tag{33}$$

where $f_2(p)$ is given by the following cumbersome integral expression:

$$f(p) = \int_0^\infty d\kappa \frac{32\kappa^2[2\kappa^2 + (\kappa^4+p^2)^2]}{[4\kappa^2 + (\kappa^2+p^2)]^{1/2}[16\kappa^4 + 8(\kappa^4+p^2) + (\kappa^4-p^2)^2]}.$$

For $p \to 0$ one can numerically find that $f(p) \to 0.33$, and for $p \gg 1$ we obtain the asymptotics $f(p) \simeq 32/\pi p$.

So far we have been dealing only with field-induced two-magnon processes. Now we shall discuss two-magnon processes due to the higher-order anisotropy, and three-magnon processes (the transformation of one magnon into two). The inclusion of the fourth-order term $\Delta w_a = b_4\sin^4\theta$ in the anisotropy energy leads to inelastic two-magnon processes, and therefore to the contribution to the viscosity that has the following form [11]:

$$\Delta\eta_2 = \frac{\hbar}{x_0^2}\begin{cases} b_1^2(p)\exp(-\hbar\omega_0/T) & T \ll \hbar\omega_0 \\ b_2^2(p)(T/\hbar\omega_0) & T \gg \hbar\omega_0 \end{cases} \tag{34}$$

where $b_{1,2}^2(p) \propto (b_4/\beta)^2 \ll 1$. Comparing (34) and (33), one can easily see that at low temperatures the field-induced viscosity contains the additional large factor $(\hbar\omega_0/T)^{1/2}$. Therefore, assuming that the small parameters ε and b_4/β_2 are of the same order of magnitude, we conclude that at low temperatures $\Delta\eta_2$ can be neglected, and that at $T \gg \hbar\omega_0$ the contributions of field-induced and anisotropy-induced processes are comparable.

The three-magnon Hamiltonian $H_{\text{int}}^{(3)}$ has the form

$$
\begin{aligned}
H_{\text{int}}^{(3)} = \sum_{1,2,3}\{&\Psi_{123}(a_1 A_2^* A_3^* + 2a_1 A_{-2} A_3^*)\exp[\mathrm{i}(k_1 - k_2 - k_3)vt] + \mathrm{cc}\} \\
&+ \sum_{1,2}\{\Psi_{12} A_0 (a_1 A_{-2}^* + a_{-1}^* A_2)\exp[\mathrm{i}(k_1 - k_2)vt] + \mathrm{cc}\} \\
&+ \sum_{k}\{\Psi_{0k} a_k A_0^* A_0^* \exp(\mathrm{i}kvt) + \mathrm{cc}\} \qquad (35)
\end{aligned}
$$

where $1 \equiv k_1$, etc. We shall not write down the expressions for the amplitudes; the only important thing to know is that the amplitudes Ψ_{123}, Ψ_{12}, and Ψ_{0k}, unlike the three-magnon amplitudes in the sine–Gordon model [10, 18], do not vanish on the mass surface of the corresponding processes (at $\omega_1 = \Omega_2 + \Omega_3$, $\omega_1 = \Omega_2 \pm \Omega_0$, and $\omega_k = 2\Omega_0$, respectively). Consequently, the five indicated processes in (35) contribute to the viscosity η.

The asymptotic form of the three-magnon contribution η_3 can be determined in the limiting cases of low and high temperatures. At $T \ll \hbar\omega_0$ the contributions from all processes are exponentially small and have the form $(T/\hbar\omega_0)^\nu \exp(-\hbar\omega_0(1+p)^{1/2}/T)$, with different rational ν (we do not give explicit expressions). At $T \gg \hbar\omega_0$ the contributions of all five three-magnon processes have the same temperature dependence:

$$\eta_3 = (T^2/E_0\omega_0 x_0^2) f_3(p) \qquad (36)$$

where $f_3(p)$ is a complicated function. It is easy to see that the quantity η_3, as compared with η_2, does not contain the small parameter ε, but contains the additional small temperature factor T/E_0. Therefore, at high temperatures η_3 can compete with η_2: for $T > E_0 h^2/\bar{\beta}_2\delta$, the contribution of η_3 is significant, $\eta \simeq \eta_3 \propto T^2$ and $D \propto 1/T$. If, however, $T < E_0 h^2/\bar{\beta}_2\delta$, then $\eta \simeq \eta_2 \propto T$ and the diffusion coefficient D does not depend on temperature down to the quantum-mechanical region $T < \hbar\omega_0$.

4. Dynamical structure factor

The DSF $S^{\alpha\beta}(q,\omega)$ is the space–time Fourier component of the spin correlation function $\langle S^\alpha(x,t)S^\beta(x',0)\rangle$, which determines the response of the magnetic system to an external action. For example, the cross section of inelastic neutron scattering is proportional to the contraction of $S^{\alpha\beta}(q,\omega)$ with some symmetrical tensor; in this case q and $\hbar\omega$ have the meaning of transferred momentum and energy, respectively. In the case of AFMs, the main contribution to the DSF originates from the correlator of the antiferromagnetism vector $\boldsymbol{l}$, and magnetization $\boldsymbol{m}$ according to (3) leads to small corrections of the order of H/H_e, where $H_e = \delta M_0/4$ is the exchange field (these corrections were investigated in [21]). Therefore, neglecting the contribution of $\boldsymbol{m}$, we can define DSF as

$$S^{\alpha\beta}(q,\omega) = \int\!\!\int \mathrm{d}x\,\mathrm{d}x' \exp[\mathrm{i}q(x-x')] \int \mathrm{d}t \exp(-\mathrm{i}\omega t)\langle l^\alpha(x,t) l^\beta(x',0)\rangle \qquad (37)$$

where q is counted from the antiferromagnetic Bragg point π/a, a being a magnetic lattice constant.

We shall be interested only in the DSF component associated with the contribution of kinks and responsible for the CP in neutron scattering experiments. The effects of soliton–magnon interference lead to corrections [22] reducing the CP intensity by a factor of about T/E_0 and will not be considered here. Assuming that the density n_s of the soliton gas is low enough, i.e., $n_s x_0 \ll 1$, we can approximate the N-kink solutions by a superposition of one-kink solutions, so that we have $l^x = 0$

$$l^y = \sum_{n=1}^{N} \sigma_n \sigma_n' \frac{1}{\cosh[(x - x_n(t))/x_0]} \tag{38}$$

$$l^z = \pm \prod_{n=1}^{N} \tanh\left[\frac{x - x_n(t)}{x_0}\right] \tag{39}$$

where $\sigma_n = \mathrm{sgn}(\cos\varphi_{0n})$, $\sigma_n' = \mathrm{sgn}(\theta_{0n}')$. It should be noted that, in writing these formulae, we have used the 'non-relativistic' limit of expression (7) since, at low temperatures $T \ll E_0$, the mean thermal velocity of a kink $v_T \ll c$.

The motion of kinks can be described by using the Langevin equation (26) for their coordinates $x_n(t)$ with some initial conditions $x_n(-\infty) = x_n^0$. There is no need to specify the initial velocities $\dot{x}_n(-\infty)$, since they are 'forgotten' during a finite time of the order of the relaxation time $\tau_r = m_*/\eta$. The averaging operation in (37) in this case includes averaging over the 'kink signs' σ_n and σ_n', over initial coordinates x_n^0 and over realizations of the random force $f(t)$. After simple transformations, we obtain the following expressions for the non-vanishing components of the DSF:

$$\begin{aligned} S^{yy}(q,\omega) &= L n_s[\pi^2/\cosh^2(\pi q x_0/2)] I(q,\omega) \\ S^{zz}(q,\omega) &= L \int\int \mathrm{d}z\,\mathrm{d}t \exp[\mathrm{i}(qz - \omega t)] \exp[-2n_s \Delta(z,t)] \end{aligned} \tag{40}$$

where n_s is the density of the kink gas, and L is the size of the system. The functions $I(q,\omega)$ and $\Delta(z,t)$ determine the shape of the CP and can be written in the form

$$\begin{aligned} I(q,\omega) &= 2\,\mathrm{Re} \int_0^{\infty} \mathrm{d}t \exp(-\mathrm{i}\omega t) \langle\langle \exp[\mathrm{i}q \Delta x_s(t)] \rangle\rangle \\ \Delta(z,t) &= \langle\langle (z - \Delta x_s(t)) \operatorname{cotanh}[(z - \Delta x_s(t))/x_0] \rangle\rangle \end{aligned} \tag{41}$$

where $\Delta x_s(t) \equiv x_s(t) - x_s(0)$; the symbol $\langle\langle \ldots \rangle\rangle$ denotes the averaging over the realizations of the random force f. Assuming for simplicity that the random process $f(t)$ is of the Gaussian type, we can express the mean values appearing in (41) only in terms of the mean-square kink displacement $\langle \Delta x_s^2(t) \rangle$. The Langevin equation (26) can be used to derive the following expression for $\langle \Delta x_s^2(t) \rangle$, which is valid for a time t much longer than the random force correlation time $\tau_{\mathrm{cor}} \simeq \omega_0^{-1}$:

$$\langle \Delta x_s^2(t) \rangle = 2Dt - 2D\tau_r (1 + D_*/D)^{-1} [1 - \exp(-t/\tau_r)] \tag{42}$$

where $\tau_r = m_*/\eta$ is the characteristic time of viscous velocity relaxation, and D and D_* are the normal anomalous diffusion coefficients. An analysis shows, however, that the effect of the D_* diffusion on the motion of a kink can be neglected. Indeed, although for $t \ll \tau_r$ we have $\langle \Delta x_s^2(t) \rangle = 2D_* t + v_T^2 t^2$, the estimates of D_* by formulae (31) show that for $t > \tau_{cor}$ the contribution of the first term is negligibly small, $2D_* t/v_T^2 t^2 < T/E_0 \ll 1$. The contribution of soliton–soliton interactions to D_* is found to be still smaller [5]. Therefore, in contrast to the opinion of the authors of [4] and [5], the D_* diffusion cannot manifest itself in the DSF at least in the region of low frequencies $\omega < \omega_0$, i.e. in the region of the CP, and in (42) we can henceforth put $D_* = 0$. Using the approximation $\langle \Delta x_s^2(t) \rangle \simeq 2D\tau_r[(1 + t^2/\tau_r^2)^{1/2} - 1]$, we can easily obtain asymptotic expressions for $I(q, \omega)$ describing the shape of the CP in two limiting cases. For $q \ll l_r^{-1}$, where $l_r = (D\tau_r)^{1/2}$ is the mean free path of a kink, we obtain the Lorentzian shape of the CP which is typical of diffusive motion:

$$I(q, \omega) \simeq 2Dq^2/[\omega^2 + (Dq^2)^2]. \tag{43}$$

For $q \gg l_r^{-1}$, we obtain the Gaussian shape corresponding to a free motion of solitons:

$$I(q, \omega) \simeq (2\pi/q^2 v_T^2)^{1/2} \exp[-\omega^2/2q^2 v_T^2]. \tag{44}$$

We shall not analyse high-frequency asymptotic forms for $\omega \gg \omega_0$, which are insignificant for describing the CP. It should be noted that expressions of the type (43) and (44) appear in any model of Brownian motion for localized components of solitons, and the only difference is in the temperature and field dependences of D. According to the formulae derived above, the CP halfwidth in the region of diffusive motion $\Gamma_\omega = Tq^2/\eta$ is temperature independent for $\hbar\omega_0 < T < E_0 h^2/\bar{\beta}_2\delta$ and is proportional to $1/T$ for $T > E_0 h^2/\bar{\beta}_2\delta$. The field dependence, according to (33) and (36), is quite complicated and is determined by the dependence on H not only of the kink parameters, but also of the functions $f_2(p)$ and $f_3(p)$ appearing in η_2 and η_3.

The calculation of the component $S^{zz}(q, \omega)$ is more complicated: the function $\Delta(z, t)$ in (41) is quite cumbersome and cannot be expressed in terms of elementary functions. However, the problem can be simplified by assuming that we are interested only in the behaviour of the DSF in the region of wavevectors q which are small as compared to the reciprocal thickness x_0^{-1} of a kink (this is the most important region, since it can be shown that $S^{zz} \simeq \exp(-qx_0)$ for $qx_0 \gg 1$). Therefore, we can put $x_0 = 0$ in (39) and obtain the following expression for $\Delta(z, t) \simeq \langle\langle |z - \Delta x_s(t)| \rangle\rangle$:

$$\Delta(z, t) \simeq [2\langle \Delta x_s^2(t) \rangle]^{1/2} \varphi\left(z/[2\langle \Delta x_s^2(t) \rangle]^{1/2}\right)$$

where

$$\varphi(u) = (2u/\pi^{1/2}) \int_0^u \mathrm{d}y \exp(-y^2) + [\exp(-u^2)]/\pi^{1/2}.$$

Using for $\varphi(u)$ the Maki approximation $\varphi(u) = (u^2 + 1/\pi)^{1/2}$ [23], we can present S^{zz} in the form of the integral

$$S^{zz}(q, \omega) = \frac{4n_s L}{(q^2 + 4n_s^2)^{1/2}} \int_0^\infty \mathrm{d}t \cos\omega t [2\langle \Delta x_s^2(t) \rangle/\pi]^{1/2} K_1 \left\{ [2\langle \Delta x_s^2(t) \rangle (q^2 + 4n_s^2)/\pi]^{1/2} \right\}$$

where $K_1(z)$ is the MacDonald function. We obtain the following asymptotic forms:

(1) $D(q^2+4n_s^2) \gg \tau_r^{-1}$. In this case, the expression for S^{zz} coincides with that obtained by Maki [23] for freely moving solitons:

$$S^{zz}(q,\omega) \simeq 4Ln_s v_T^2/[\omega^2 + 2v_T^2(q^2+4n_s^2)/\pi]^{3/2}. \tag{45}$$

(2) $D(q^2+4n_s^2) \ll \tau_r^{-1}$. For small wavevectors, we have a cumbersome expression for S^{zz} in terms of the Whittaker function $W_{-1;1/2}(z)$ [24]:

$$S^{zz}(q,\omega) \simeq [4Ln_s/(q^2+4n_s^2)] \times \mathrm{Re}\{\mathrm{i}\exp[\mathrm{i}D(q^2+4n_s^2)/2\pi|\omega|]W_{-1;1/2}[\mathrm{i}D(q^2+4n_s^2)/\pi|\omega|]\}. \tag{46}$$

For high and low frequencies, this formula can be written as

$$S^{zz}(q,\omega) \simeq [4\pi n_s L/D(q^2+4n_s^2)][1 - 6\pi^2\omega^2/D^2(q^2+4n_s^2)^2] \qquad |\omega| \ll D(q^2+4n_s^2)$$

$$S^{zz}(q,\omega) \simeq (4Dn_s/\pi\omega^2)\ln|\pi\omega/D(q^2+4n_s^2)| \qquad |\omega| \gg D(q^2+4n_s^2).$$

It hence follows that (46) describes a 'diffusive' CP with a halfwidth $\Delta\omega$ of the order of $D(q^2+4n_s^2)$. The fact that $\Delta\omega$ is finite for $q=0$ is determined by the finite density of kinks and by kink recharging effects during their collisions [5]. Recharging for S^{zz} is taken into account explicitly by using the expression (39). It should be noted that if we try to take into account these effects for the transverse component S^{yy}, the CP halfwidth for this component also becomes finite at $q=0$, which can be qualitatively described by the substitution $Dq^2 \to D(q^2+4n_s^2)$ in formula (43).

5. Conclusion

The analysis of the vector model of AFMs carried out in this paper shows that this model is characterized by a more complicated behaviour than the conventional sine–Gordon model, and exhibits a number of effects. In particular, normal Einsteinian diffusion of kinks appears due to the non-integrability of the vector model, which in turn leads to considerable modification of the soliton CP in the region of small wavevectors. We have analysed both two- and three-magnon processes, and found that their contributions to the diffusion coefficient can compete; the 'weight' of the two-magnon contribution can be controlled by the external magnetic field, which gives an interesting possibility of distinguishing between two- and three-magnon processes in neutron experiments.

Acknowledgments

The authors are grateful to V G Bar'yakhtar for fruitful discussions. This work was supported, in part, by grant No 2/361 from the Science and Technology Committee of the Ukraine. Two of us (BI and AK) were also supported by a Soros Foundation Grant awarded by the American Physical Society.

References

[1] Trullinger S E, Zakharov V E and Pokrovskiĭ V L (ed) 1986 *Solitons* (Amsterdam: Elsevier)
[2] Izyumov Yu A 1988 *Sov. Phys.–Usp.* **31** 689
[3] Mikeska H J 1980 *J. Phys. C: Solid State Phys.* **13** 2913
[4] Boucher J P, Regnault L P, Pynn R, Bouillot J and Renard J P 1986 *Europhys. Lett.* **1** 415
[5] Sasaki K and Maki K 1987 *Phys. Rev.* B **35** 257
[6] Ivanov B A and Kolezhuk A K 1990 *Phys. Lett.* **146A** 190
[7] Wada Y and Schrieffer J R 1978 *Phys. Rev.* B **18** 3897
[8] Bar'yakhtar V G, Bar'yakhtar I V, Ivanov B A and Sukstanskiĭ A L 1978 *Preprint* D17-81-78 Joint Inst. Nucl. Res., Dubna
[9] Kunz C 1986 *Phys. Rev.* A **34** 510
[10] Ogata M and Wada Y 1986 *J. Phys. Soc. Japan* **55** 1252
[11] Ivanov B A, Sukstanskiĭ A L and Tartakovskaya E V 1987 *Sov. J. Low Temp. Phys.* **13** 560
[12] Bar'yakhtar I V and Ivanov B A 1979 *Sov. J. Low Temp. Phys.* **5** 361
[13] Ivanov B A, Kolezhuk A K and Oksyuk G K 1991 *Europhys. Lett.* **14** 151
[14] Ivanov B A and Kolezhuk A K 1991 *Sov. J. Low Temp. Phys.* **17** 177
[15] Wysin G M, Bishop A R and Oitmaa J 1986 *J. Phys. C: Solid State Phys.* **19** 221
[16] Ivanov B A and Sukstanskiĭ A L 1988 *Sov. Phys.–JETP* **67** 1201
[17] Dzyub I P and Zerov Yu 1988 *Preprint* ITP-88-54E Inst. Theor. Phys., Kiev
[18] Bar'yakhtar V G, Ivanov B A, Sukstanskiĭ A L and Tartakovskaya E V 1988 *Teor. Mat. Fiz.* **74** 46 (in Russian)
[19] Akhiezer A I and Peletminskiĭ S V 1977 *Methods of Statistical Physics* (Moscow: Nauka) (in Russian)
[20] Ivanov B A and Kolezhuk A K 1989 *JETP Lett.* **49** 561
[21] Sasaki K 1984 *J. Phys. Soc. Japan* **53** 2872
[22] Allroth E and Mikeska H J 1981 *Z. Phys.* B **43** 209
[23] Maki K 1980 *J. Low Temp. Phys.* **41** 327
[24] Gradshtein I S and Ryzhik I M 1965 *Tables of Integrals, Sums, Series, and Products* 4th edn (New York: Academic)